Math 113 (Calculus 2) Exam 4

November 20 – November 24, 2009
Sections 1–10, 13–17

Name ______________________
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Section ______________________
Instructor ______________________

In some cases a series may be seen to converge or diverge for more than one reason. For such problems choose the answer that is easiest to understand. For example, don't choose that a series converges by the Integral Test if you get an integral that is too difficult to evaluate, or don't choose the Root Test if the limit of the roots is too difficult to evaluate.

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$

   (a) The series converges absolutely by the Integral Test.
   (b) The series diverges by the Integral Test.
   (c) The series converges conditionally.
   (d) The series diverges by the Test for Divergence.

2. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

   (a) The series converges absolutely.
   (b) The series converges conditionally.
   (c) The series diverges by the Test for Divergence.
   (d) The terms go to zero, but the series still diverges.

3. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

   (a) The series converges by the Integral Test.
   (b) The series diverges by the Integral Test.
   (c) The series converges by the Ratio Test.
   (d) The series diverges by the Ratio Test.
   (e) The series converges by the Test for Divergence.
   (f) The series diverges by the Test for Divergence.

4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n^{1.2}}$

   (a) The series converges absolutely.
   (b) The series converges conditionally.
   (c) The series diverges by the Test for Divergence.
   (d) The terms go to zero, but the series still diverges.

5. $\displaystyle\sum_{k=1}^{\infty} \frac{k+1}{\sqrt{k^5}}$

(a) The series converges by the Comparison Test with $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$.

(b) The series diverges by the Comparison Test with $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$.

(c) The series converges by the Limit Comparison Test with $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$.

(d) The series diverges by the Limit Comparison Test with $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$.

(e) The series diverges by the Test for Divergence.

(f) The terms go to zero, but the series still diverges.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n+1)}$

(a) The series diverges by the Ratio Test because the limit ratio is 2.

(b) The Ratio Test gives no information.

(c) The series converges by the Ratio Test because the limit ratio is $\dfrac{1}{2}$.

(d) The series converges by the Ratio Test because the limit ratio is $\dfrac{1}{3}$.

(e) The series converges by the Ratio Test because the limit ratio is 0.

7. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(a) The series diverges by the Ratio Test because the limit ratio is e

(b) The Ratio Test gives no information.

(c) The series converges by the Ratio Test because the limit ratio is 0.

(d) The series converges by the Ratio Test because the limit ratio is $\dfrac{1}{e}$.

(e) The series converges by the Integral Test.

8. $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k^3+1}{2k^3+3}\right)^{2k}$

(a) The series diverges by the Ratio Test.

(b) The series diverges by the Root Test.

(c) The Root Test gives no information.

(d) The series converges by the Root Test because the limit root is $1/2$

(e) The series converges by the Root Test because the limit root is $1/4$.

(f) The series converges by the Root Test because the limit root is $1/9$.

9. $\displaystyle\sum_{m=1}^{\infty} \sin\left(\frac{1}{m^2}\right)$

(a) The series diverges by the Test for Divergence.

(b) The series converges by the Integral Test.

(c) The series converges by the Ratio Test.

(d) The series converges by the Root Test.

(e) The series converges by the Limit Comparison Test with $\displaystyle\sum_{m=1}^{\infty} \frac{1}{m^2}$

10. Find the coefficient of x^{1000} in the MacLaurin series for $x^2 \sin x^2$.

(a) $\frac{1}{1000!}$ (b) $\frac{-1}{1000!}$ (c) $\frac{1}{999}$ (d) $\frac{-1}{999!}$ (e) $\frac{1}{500!}$ (f) $\frac{-1}{500!}$ (g) $\frac{1}{499!}$ (h) $\frac{-1}{499!}$

11. Evaluate the sum. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$.

(a) $\frac{3}{2}$ (b) $\frac{7}{4}$ (c) $\frac{17}{8}$ (d) 2 (e) e (f) $\frac{\pi}{2}$ (g) $\frac{3\pi}{2}$ (h) 3

12. Evaluate the sum $\displaystyle\frac{\pi}{6} - \frac{\pi^3}{6^3 3!} + \frac{\pi^5}{6^5 5!} - \frac{\pi^7}{6^7 7!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{6^{2n+1}(2n+1)!}$

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{3}{2}$ (f) $\frac{5}{2}$ (g) $\frac{7}{2}$ (h) $\frac{\sqrt{3}}{2}$

13. Evaluate the sum. $\displaystyle x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots = \sum_{n=1}^{\infty} \frac{x^n}{n}$

(a) $-\ln(1+x)$ (b) $-\ln(1-x)$ (c) $\ln(1+x)$ (d) $\ln(1-x)$ (e) $\frac{1}{1-x}$

14. Evaluate the sum. $\displaystyle x^2 + x^4 + \frac{x^6}{2!} + \frac{x^8}{3!} + \frac{x^{10}}{4!} + \frac{x^{12}}{5!} + \cdots = \sum_{k=1}^{\infty} \frac{x^{2k}}{(k-1)!}$

(a) $x^2 e^x$ (b) xe^{x^2} (c) $x^2 e^{x^2}$ (d) $\frac{x^2}{1-x^2}$ (e) $\sin(2x)$

15. Evaluate the following limit: $\displaystyle\lim_{x\to 0} \frac{3\tan^{-1} x^2 - 3x^2 + x^6}{x^{10}}$

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$ (e) $\frac{3}{5}$ (f) $-\frac{3}{5}$ (g) $\frac{1}{4}$ (h) $-\frac{1}{4}$

16. Find the the coefficient of x^6 in the power series expansion for the function $\dfrac{1}{\sqrt[3]{1+x^2}}$ expanded about $x = 0$.

(a) $-\dfrac{14}{9}$ (b) $\dfrac{14}{9}$ (c) $-\dfrac{14}{27}$ (d) $\dfrac{14}{27}$ (e) $-\dfrac{14}{81}$ (f) $\dfrac{14}{81}$

17. Find the radius of convergence. $\displaystyle\sum_{n=1}^{\infty} (-1)^n 2^n n^3 x^n$

(a) 0 (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{\sqrt{2}}$ (d) 2 (e) $\sqrt{2}$ (f) ∞

18. Find the radius of convergence. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n x^{2n}}{n}$

(a) 0 (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{\sqrt{2}}$ (d) 2 (e) $\sqrt{2}$ (f) ∞

19. Find the coefficient of x^4 in the MacLaurin series for $e^x \cos x^2$.

(a) $\dfrac{13}{24}$ (b) $-\dfrac{13}{24}$ (c) $\dfrac{11}{24}$ (d) $-\dfrac{11}{24}$ (e) $\dfrac{15}{24}$

20. Find the fourth non-zero term for the Taylor series for $\cos x$ about $x = \pi/2$.

(a) $\dfrac{(x-\pi/2)^7}{7!}$

(b) $-\dfrac{(x-\pi/2)^7}{7!}$

(c) $\dfrac{(x-\pi/2)^9}{9!}$

(d) $-\dfrac{(x-\pi/2)^9}{9!}$

21. Find the Taylor series for e^{2x} about $x = \ln 2$.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n (x-\ln 2)^n}{n!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+1} (x-\ln 2)^n}{n!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n (x-\ln 2)^{n+1}}{n!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+2} (x-\ln 2)^n}{n!}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n (x-\ln 2)^{n+2}}{n!}$

(f) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+1} (x-\ln 2)^{n+1}}{n!}$

22. Find the Maclaurin series for $\frac{1}{x^2+2}$.

(a) $\frac{1}{2}(1 - \frac{x^2}{2^2} + \frac{x^4}{2^4} - \frac{x^6}{2^6} + \cdots)$

(b) $\frac{1}{2}(1 + \frac{x^2}{2^2} + \frac{x^4}{2^4} + \frac{x^6}{2^6} + \cdots)$

(c) $\frac{1}{2}(1 - \frac{x^2}{2} + \frac{x^4}{2^2} - \frac{x^6}{2^3} + \cdots)$

(d) $\frac{1}{2}(1 + \frac{x^2}{2} + \frac{x^4}{2^2} + \frac{x^6}{2^3} + \cdots)$

23. Power series considerations show that $\int_0^1 e^{-x^2}\,dx = 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} \cdots$. What is the next term in the series?

(a) $\frac{1}{120}$ (b) $\frac{1}{126}$ (c) $\frac{1}{132}$ (d) $\frac{1}{196}$ (e) $\frac{1}{216}$ (f) $\frac{1}{252}$ (g) $\frac{1}{296}$

24. If $f(x) = x \sin x$, find the 100th derivative evaluated at zero; i.e., find $f^{(100)}(0)$.

(a) $100!$ (b) $-100!$ (c) $\frac{1}{99!}$ (d) $-\frac{1}{99!}$ (e) 100 (f) -100 (g) 0

25. Suppose f is a function so that its fourth derivative satisfies $-3 \leq f^{(4)}(x) \leq 2$ for all values of x. If $T_3(x)$ is the third-degree polynomial of f centered at 5, then Taylor's Inequality says that the error when using $T_3(6)$ to approximate $f(6)$ is less than or equal to:

(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ (e) $\frac{1}{4}$ (f) $\frac{1}{8}$ (g) $\frac{1}{12}$

1. c
2. b
3. b
4. a
5. c
6. c
7. d
8. e
9. e
10. h
11. d
12. d
13. b
14. c
15. e
16. e
17. b
18. b
19. d
20. a
21. d
22. c
23. e
24. f
25. f